The fault detection and identification problem as a dual case of the disturbance rejection.*

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Abstract

In this paper we present a new result concerning the Failure Detection and Identification problem as a straightforward interpretation of the dual problem: the input-output disturbance rejection problem. This result leads in a natural and simple way to the classical result of Massoumia.

1 Introduction

The problem of failure detection and identification is always present when dealing with modern technological systems in order to satisfy increasingly stringent safety and environmental regulations Monitoring pursuits the minimization of risk, which is in fact unavoidable. Failures in actuators, sensors and components are normal sources of risk in technological systems and because of this fault detection, identification and isolation systems are common in monitoring processes. When monitored technological systems are described in terms of integral-differential equations, model based fault detection and isolation can be achieved (see [6]). These techniques minimizes the complexity of sensors systems, which are necessary in order to reduce the associated cost.

The problem of fault detection and identification in dynamical systems consist in the generation of signals sensitive to the generation of some particular faults. We tackle here the deterministic time invariant linear case concerning the residual generation (full order observers involved). Massoumia ([2], [4] and [3]) uses the so-called Beard and Jones filter to give geometric conditions to solve the problem of residual generation. In this paper we will serve us of classical results on the disturbance rejection by state feedback and the dual case, as stated in [1], in order to obtain a simple solution to a new verion of the so called Fundamental Problem of Residual Generation. Then, in a natural way, the result of Massoumia [4] is obtained.

The paper is organized as follows. In Section 2 we introduce the notation and recall the definition of some useful invariant subspaces. Section 3 is dedicated to recall the disturbance rejection problem by (static) state feedback and the dual case. In Section4 We present our contribution: geometric conditions for the existence of a solution to the Fault Detection and Isolation problem. In Section 5 two illustrative examples are presented and finally, in Section 6 some concluding remarks are given.

2 Preliminaries: some useful subspaces

Consider a linear time-invariant system $(A,\ B,\ C)$ described by:

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{array} \right. \tag{1}$$

where: $x(t) \in \mathcal{X} \simeq \mathsf{R}^n$ denotes the state; $u(t) \in \mathcal{U} \simeq \mathsf{R}^m$ denotes the input, and $y(t) \in \mathcal{Y} \simeq \mathsf{R}^p$ the output. It is considered here that $A : \mathcal{X} \to \mathcal{X}$, B :

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$\mathcal{U} \to \mathcal{X}$, and $C : \mathcal{X} \to \mathcal{Y}$, are linear maps represented by real constant matrices.

Consider a subspace $\mathcal{K} \subseteq \mathcal{X}$ be given. The notation:

- $\mathcal{V}_{(\mathcal{K})}$ indicates that the subspace $\mathcal{V}_{(\mathcal{K})}$ is $(A, \mathcal{K})$-invariant,i.e., $A\mathcal{V}_{(\mathcal{K})} \subseteq \mathcal{V}_{(\mathcal{K})} + \mathcal{K}$. We note $\mathcal{F}\left(\mathcal{V}_{(\mathcal{K})}\right)$ the family of maps $F : \mathcal{X} \to \mathcal{U}$ (state feedbacks) such that $(A + BF)\,\mathcal{V}_{(\mathcal{K})} \subseteq \mathcal{V}_{(\mathcal{K})}$.
- $\mathcal{S}_{(\mathcal{K})}$ indicates that the subspace $\mathcal{S}_{(\mathcal{K})}$is $(\mathcal{K}, A)$-invariant,i.e., $A\left(\mathcal{K} \cap \mathcal{S}_{(\mathcal{K})}\right) \subseteq \mathcal{S}_{(\mathcal{K})}$. We note $\mathcal{D}\left(\mathcal{S}_{(\mathcal{K})}\right)$ the family of maps $G : \mathcal{Y} \to \mathcal{X}$ (output injections) such that $(A + GC)\,\mathcal{S}_{(\mathcal{K})} \subseteq \mathcal{S}_{(\mathcal{K})}$.

We say that $\mathcal{R}_{(\operatorname{Im} B)}$ is an $(A, \operatorname{Im} B)$ controllability subspace if there exists a couple of maps $F : \mathcal{X} \to \mathcal{U}$ and $\theta : \mathcal{U} \to \mathcal{U}$ such that the subspace $\mathcal{R}_{(\operatorname{Im} B)}$ satisfies $\mathcal{R}_{(\operatorname{Im} B)} = \langle A + BF \mid \operatorname{Im}(B\theta)\rangle := \operatorname{Im}(B\theta) + (A + BF)\operatorname{Im}(B\theta) + \cdots + (A + BF)^{n-1}\operatorname{Im}(B\theta)$.

We say that $\mathcal{N}_{(\operatorname{Ker} C)}$ is $(\ker C, A)$-complementary observability (or simply $(\ker C, A)$-unobservability) subspace if there exists a couple of maps $D : \mathcal{Y} \to \mathcal{X}$ and $H : \mathcal{Y} \to \mathcal{Y}$ such that the subspace $\mathcal{N}_{(\operatorname{Ker} C)}$ satisfies $\mathcal{N}_{(\operatorname{Ker} C)} = \langle \operatorname{Ker}(HC) \mid A + DC\rangle := \operatorname{Ker}(HC) \cap (A + DC)^{-1}\operatorname{Ker}(HC) \cap \cdots \cap (A + DC)^{-n+1}\operatorname{Ker}(HC)$.

Given two subspaces $\mathcal{K}$ and $\mathcal{L} \subseteq \mathcal{X}$, we shall note (see [7, Wonham, 1985] and [1, Basile and Marro, 1992]):

- $\mathcal{V}^*_{(\mathcal{K},\mathcal{L})}$:=the supremal $(A, \mathcal{K})$-invariant subspace $(\mathcal{V}_{(\mathcal{K})})$contained in $\mathcal{L}$;
- $\mathcal{S}^*_{(\mathcal{L},\mathcal{K})}$:=the infimal $(\mathcal{L}, A)$-invariant subspace $(\mathcal{S}_{(\mathcal{L})})$ containing $\mathcal{K}$;
- $\mathcal{R}^*_{(\mathcal{K},\mathcal{L})}$:= the supremal $(A, \mathcal{K})$-controllability subspace $(\mathcal{R}_{(\mathcal{K})})$ contained in $\mathcal{L}$ $(\mathcal{R}^*_{(\mathcal{K},\mathcal{L})} = \mathcal{V}^*_{(\mathcal{K},\mathcal{L})} \cap \mathcal{S}^*_{(\mathcal{L},\mathcal{K})})$;
- $\mathcal{N}^*_{(\mathcal{L},\mathcal{K})}$:=the infimal $(\mathcal{L}, A)$-unobservability subspace $\mathcal{N}_{(\mathcal{L})}$ containing $\mathcal{K}$ $(\mathcal{N}^*_{(\mathcal{L},\mathcal{K})} = \mathcal{V}^*_{(\mathcal{K},\mathcal{L})} + \mathcal{S}^*_{(\mathcal{L},\mathcal{K})})$.

$\sigma\left(F_{\mathcal{L}} \mid \mathcal{L}/\mathcal{M}\right)$ denotes the spectrum of the map induced by $(A + BF_{\mathcal{L}})$ in the quotient space $\frac{\mathcal{L}}{\mathcal{M}}$, where $\mathcal{M} \subset \mathcal{L}$ and both are $(A, \mathcal{B})$-inv.sps. The spectrum of $(A + BF_{\mathcal{V}})$ can be decomposed (in connection with the $(A, \mathcal{B})$-inv.sp. $\mathcal{V}$) into fixed and free parts (see [5]). The fixed part (called the fixed spectrum of $\mathcal{V}$) is given by:

$$\begin{aligned}\sigma_{fix}(\mathcal{V}) \quad : \quad &= \sigma\left(F_{\mathcal{V}} \mid \mathcal{X}/\langle A \mid \mathcal{B}\rangle + \mathcal{V}\right) \\ &\dot{\cup}\, \sigma\left(F_{\mathcal{V}} \mid \mathcal{V}/\mathcal{R}^*_{(\mathcal{B},\mathcal{V})}\right) \end{aligned} \tag{2}$$

for any $F_{\mathcal{V}}$ and where $\dot{\cup}$ stands for the union of sets with common elements repeated. The set $\sigma\left(F_{\mathcal{V}} \mid \mathcal{V}/\mathcal{R}^*_{(\mathcal{B},\mathcal{V})}\right)$ is called the internal fixed spectrum of $\mathcal{V}$ and $\sigma\left(F_{\mathcal{V}} \mid \mathcal{X}/\langle A \mid \mathcal{B}\rangle + \mathcal{V}\right)$ the external fixed spectrum of $\mathcal{V}$. The internal fixed spectrum of $\mathcal{V}^*_{(\mathcal{B},\mathcal{E})}$ correspond to the so-called *invariant zeros* of (A, B, E) (see [1]), i.e.:

$$\mathcal{Z}(A, B, E) := \sigma\left(F_{\mathcal{V}^*_{(\mathcal{B},\mathcal{E})}} \mid \mathcal{V}^*_{(\mathcal{B},\mathcal{E})}/\mathcal{R}^*_{(\mathcal{B},\mathcal{E})}\right)$$

Note that in the same way, the invariant zeros of the systems (A, B, C), $(A, \begin{bmatrix} B \vdots D \end{bmatrix}, E)$, ... are well defined.

In a dual way, $\sigma\left(G_{\mathcal{M}} \mid \mathcal{L}/\mathcal{M}\right)$, the spectrum of the map induced by $(A + G_{\mathcal{M}}C)$ in the quotient space $\frac{\mathcal{L}}{\mathcal{M}}$ is defined and can be decomposed (in connection with the $(\mathcal{C}, A)$-inv.sp. $\mathcal{S}$) into fixed and free parts in order to get $\sigma_{fix}(\mathcal{S})$ and the *invariant zeros* of (A, B, E) can also be obtained in a dual equivalent way:

$$\mathcal{Z}(A, B, E) := \sigma\left(G_{\mathcal{S}^*_{(\mathcal{E},\mathcal{B})}} \mid \mathcal{N}^*_{(\mathcal{E},\mathcal{B})}/\mathcal{S}^*_{(\mathcal{E},\mathcal{B})}\right)$$

(see [1]), and in the same way, the invariant zeros of the systems (A, D, E), $(A, D, \begin{bmatrix} E \\ C \end{bmatrix})$, ... are well defined.

3 Some related results on the disturbance rejection problem.

Let us consider the continuous linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Lm(t), \\ y(t) = Cx(t) \\ z(t) = Ex(t), \end{cases}$$

were $x(t), u(t)$ and $C(t)$ are defined as in 1, $m(t)$ denotes an input disturbance and $z(t)$ the interest output. We note $\mathcal{L} = imL$, $\mathcal{B} = imB$, $\mathcal{C} = \ker L$, $\mathcal{E} = \ker E$,

If we are interested in the disturbance rejection problem, i.e., in decoupling the disturbance $m(t)$ at the output $z(t)$ by static state feedback (DRSF

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problem), the necessary and sufficient condition is very well known (see [7], [1]):

$$\mathcal{L} \subset \mathcal{V}^*_{(\mathcal{B},\mathcal{E})}.$$

If this conditions holds, then there exist a $F \in \mathcal{F}\left(\mathcal{V}^*_{(\mathcal{B},\mathcal{E})}\right)$ such that,

$$\frac{Z(s)}{Q(s)} = E\left(sI - A - BF\right)^{-1} L = 0.$$

A dual way to solve the Disturbance Rejection Problem is to use an output injection strategy instead of a state feedback control, i.e., to find a G such that

$$\frac{Z(s)}{Q(s)} = E\left(sI - A - GC\right)^{-1} L = 0.$$

This is the so called Disturbance Rejection by (static) Output Injection (DROI). The necessary and sufficient condition to solve this problem is

$$\mathcal{S}^*_{(\mathcal{C},\mathcal{L})} \subset \mathcal{E} \qquad (3)$$

If this condition holds, the DROI problem can be solved with $G \in \mathcal{G}(\mathcal{S}^*_{(\mathcal{C},\mathcal{L})})$.

A different interpretation of this last result is the following: equation (3) is the necessary and sufficient condition for the existence of an observer to reconstruct the signal $z(t) = Ex(t)$ when the (disturbance) input $m(t)$ is not measured (see [1]). This interpretation of the DROI problem is very important in order to solve the Failure Detection problem, as we will see in the next section.

4 The Fault Detection and Identification problem.

Consider a continuous linear time-invariant system which includes an actuator failures model (A, B, C, $\begin{bmatrix} L_1 & L_2 & \cdots & L_l \end{bmatrix}$) described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + \sum_{i=1}^{l} L_i m_i(t), \\ y(t) = Cx(t) \\ z(t) = Ex(t), \end{cases} \qquad (4)$$

where: $m_i(t) \in \mathcal{M}_i \simeq \mathsf{R}$ denotes the i-th actuator failure mode and $L_i : \mathcal{M}_i \to \mathcal{X}$ denotes the i-th actuator failure signature.

Remark 1 *The unknown i-th actuator failure mode $m_i(t)$ has the following property:*

$$m_i(t) \neq 0$$

when the i-th actuator is failing, if it is not the case $m_i(t) = 0$.

We shall note

$$\overline{m_i}(t) =$$

$$= [m_1(t) \vdots m_2(t) \vdots \ldots \vdots m_{i-1}(t) \vdots m_{i+1]}(t) \ldots \vdots m_l(t)]$$

and $\overline{\mathcal{L}_i} = Im\overline{L_i} := \Sigma_{j \neq i} \operatorname{Im} L_j$.

The Fundamental Problem of Residual Generation (FPRG) is defined in the following way:

Definition 2 FPRG*: Get a processor that takes $y(t)$ and $u(t)$ as inputs and generates the residual $r_i(t)$ such that this $r_i(t)$ is affected by $m_i(t)$ but not by $\overline{m_i}(t)$.*

We know from Section 3 that $\mathcal{S}^*_{(\mathcal{C},\mathcal{L})} \subset \mathcal{E}$ is the necessary and sufficient condition for the existence of an observer to reconstruct the signal $z(t) = Ex(t)$ when the (disturbance) input $m(t)$ is not measured (see [1]). Then if the signal $z(t) = Ex(t)$ is available and the map E is such that $\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})} \subset \ker E$, we can build an observer providing an adequate estimated of $z(t)$, i.e., $\widehat{z}(t) = E\widehat{x}(t)$. and the residual $r_i(t) = z(t) - \widehat{z}(t) = 0$ even in the case $\overline{m_i}(t) \neq 0$. If we add the condition $\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})} \cap \mathcal{L}_i = 0$, then $r \neq 0$ when $m_i(t) \neq 0$ This interpretation of the DROI problem provides a first solution to the FPRG as follows.

Lemma 3 *Consider system (4) and the fact that the function $z(t) = Ex(t)$ is available. Then the FPRG has a solution iff*

$$\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})} \cap \mathcal{L}_i = 0, \qquad (5)$$

where the map E is such that $\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})} \subset \ker E$.

The details of the proof is omitted here, however, it is almost direct from the results given in Section 3.

This result is not of a practical interest because of the $Ex(t)$ availability restriction. However, it allows to understand the problem and to get the practical solution in a simple and natural way.

Note that there exist a set of fixed poles associated to the solution $\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})}$, i.e., the set $\sigma_{fix}(\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})})$. However, this set do not represent the fixed poles of the problem and only the fixed poles associated to the solution $\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})}$.

In a practical situation, we are restricted to the information contained in $y(t) = Cx(t)$, i.e., $z(t) = Ex(t)$ is not available. From the definition of ($\ker C$, A)-unobservability subspace $\mathcal{N}_{(\mathrm{Ker}C)} = \langle \mathrm{Ker}\,(HC) \mid A + DC \rangle := \mathrm{Ker}(HC) \cap (A+DC)^{-1}\mathrm{Ker}(HC) \cap \cdots \cap (A+DC)^{-n+1}\mathrm{Ker}(HC)$, we can see that if there

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exist a $(\ker C, A)$-unobservability subspace $\mathcal{N}_{(\mathrm{Ker}C)}$ such that

$$\overline{\mathcal{L}_i} \subset \mathcal{N}_{(\mathrm{Ker}C)}$$
$$and$$
$$\mathcal{N}_{(\mathrm{Ker}C)} \cap \mathcal{L}_i = 0$$

then the FPRG as a solution. From the definition of $\mathcal{N}^*_{(\mathcal{L},\mathcal{K})}$ the infimal $(\mathcal{L}, A)$-unobservability subspace containing a subspace $\mathcal{K}$ we can easily obtain the following result and with out need of any proof.

Theorem 4 *[4]Consider system (4). Then the FPRG has a solution iff*

$$\mathcal{N}^*_{(\mathcal{C},\overline{\mathcal{L}_i})} \cap \mathcal{L}_i = 0.$$

Note that as $\mathcal{S}^*_{(\mathcal{C},\overline{\mathcal{L}_i})} \subset \mathcal{N}^*_{(\mathcal{C},\overline{\mathcal{L}_i})}$, this condition is obviously more restrictive than (5).

Then $r_i(t)$ is affected by $m_i(t)$ but not by $\overline{m_i}(t)$, with $r_i(t) = HCx(t) - HC\widehat{x}(t)$ with H such that

$$\mathcal{N}^*_{(\mathcal{C},\mathcal{L})} \subset \ker HC$$

An important observation is that using the subspace $\mathcal{N}^*_{(\mathcal{C},\overline{\mathcal{L}_i})}$ as a geometric support for the solution, they are not fixed poles associated to the FPRG, but the non observable set, i.e. $\langle \mathrm{Ker}C \mid A\rangle$. This is obvious from the fact that

$$\sigma_{fix}(\mathcal{N}^*_{(\mathcal{C},\overline{\mathcal{L}_i})}) = \sigma(\langle \mathcal{C} \cap \ker E \mid A\rangle) \dot{\cup} \{\oslash)\}$$

5 Examples

5.1 Example 1

In order to illustrate the application of the previous result, we present now a simple example.

Consider a linear time-invariant system (A, B, C, E, L_1, L_2) described by 4, with:

$$A = \begin{bmatrix} 0 & 3 & 4 \\ -1 & -2 & -3 \\ 0 & 2 & 5 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$
$$L_1 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}; L_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 1 & 3 & 0 \end{bmatrix}$$

We are interested in a processor that takes $y(t)$ and $u(t)$ as inputs and generates a residual $r(t)$ affected by $m_2(t)$ but not by $m_1(t)$.It is easy to verify that $\mathcal{L}_1 = \mathcal{S}^*_{(\mathcal{C},\mathcal{L}_1)}$ and $\mathcal{S}^*_{(\mathcal{C},\mathcal{L}_1)} \cap \mathcal{L}_2 = 0$. As $\mathcal{S}^*_{(\mathcal{C},\mathcal{L}_1)} = \ker E$. Then conditions of Lemma 3 are satisfied. We can then build the observer

$$\begin{cases} \dot{\widehat{x}}(t) = A\widehat{x}(t) + Bu(t) + GC[\widehat{x}(t) - x(t)], \\ y(t) = C\widehat{x}(t), \end{cases}$$

with $G \in \mathcal{S}^*_{(\mathcal{C},\mathcal{L}_1)}$. Let us consider

$$G = \begin{bmatrix} 0 & -2 \\ -2 & -1 \\ -2 & -9 \end{bmatrix}$$

It is easy to check that the residual generator $r_2(t) = Ex(t) - E\widehat{x}(t)$ is sensitive to $m_2(t)$ but not to $m_1(t)$, i.e.,

$$\frac{R_2(s)}{M_1(s)} = 0$$

and

$$\frac{R_2(s)}{M_2(s)} \neq 0$$

Note that the restriction $G \in \mathcal{N}^*_{(\mathcal{C},\mathcal{L}_1)}$ impose the pole $s = -3$, i.e., $\sigma_{fix}(\mathcal{S}^*_{(\mathcal{C},\mathcal{L}_1)}) = \{-3\}$.

5.2 Example 2

Let us consider the (A, B, C, E, L_1, L_2) system, with matrices A, B, C, E and L_1 as in Example 1 but with L_2 as follows:

$$L_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

It is easy to check that

$$\mathcal{N}^*_{(\mathcal{C},\mathcal{L}_1)} = \left\{ \begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} \right\}$$

As

$$\mathcal{N}^*_{(\mathcal{C},\mathcal{L}_1)} \cap imL_2 = 0$$

Then conditions of Theorem 4 are satisfied. We can then build the observer

$$\begin{cases} \dot{\widehat{x}}(t) = A\widehat{x}(t) + Bu(t) + GC[\widehat{x}(t) - x(t)], \\ y(t) = C\widehat{x}(t), \end{cases}$$

with $G \in \mathcal{N}^*_{(\mathcal{C},\mathcal{L}_1)}$. Let us consider

$$G = \begin{bmatrix} 1 & -2 \\ -2 & -1 \\ -2 & -9 \end{bmatrix}$$

It is easy to check that the residual generator $r_2(t) = HCx(t) - HC\widehat{x}(t)$, with $H = \begin{bmatrix} 0 & 1 \end{bmatrix}$, is sensitive to $m_2(t)$ but not to $m_1(t)$, i.e.,

$$\frac{R_2(s)}{M_1(s)} = 0$$

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and

$$\frac{R_2(s)}{M_2(s)} \neq 0$$

Note that the restriction $G \in \mathcal{N}^*_{(\mathcal{C},\mathcal{L}_1)}$ do not impose any pole, i.e., all the poles can be freely placed in the observer with $G \in \mathcal{N}^*_{(\mathcal{C},\mathcal{L}_1)}$.

6 Concluding Remarks.

In this paper we has presented a new result concerning the Failure Detection and Identification Problem. A simple necessary and sufficient condition to solve one version of the failure detection and identification problem was obtained. The results are stated as a straightforward interpretation of the dual problem of the input-output disturbance rejection problem, i.e., the estimation of a linear function of the state when the input is not measured. Even if this result is not of a practical interest, it allows to understand the problem and to obtain the practical results of Massoumia [4] in a sample and direct way.

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